MR0232807 (38 #1130) 17.30
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On the theory of soluble Lie algebras.
Math. Z. **106** 1968 343–354

In recent years the theory of formations and homomorphs of finite soluble groups [cf. W. Gaschütz, same Z. **80** (1962/63), 300–305; MR0179257; H. Schunck, ibid. **97** (1967), 326–330; MR0209356] has produced new ideas for finding interesting characteristic classes of conjugate sub-groups of finite soluble groups. In this interesting paper this theory is translated to the theory of soluble Lie algebras of finite dimension over some commutative field. The question of conjugacy of the subalgebras produced by this theory is dealt with (for the problems involved, cf. also the first author's paper ibid. **101** (1967), 350–355 [MR0220785]). The upshot of all this is that if the ground field is not algebraically closed, the various ways to define, e.g., saturated formations in finite soluble groups, turn out to yield distinct results; while over an algebraically closed ground field of characteristic 0, the theory goes through as in the group case, but the sparsity of saturated homomorphs here shows that in this class of Lie algebras one does not gain by studying formations and homomorphs: The only saturated homomorphs of finite-dimensional soluble Lie algebras over an algebraically closed field of characteristic 0 are the class consisting of 0 alone, the nilpotent algebras, and the soluble algebras (which are supersolvable). In this language, one also obtains an easy characterisation of the Cartan and Borel subalgebras of an arbitrary finite-dimensional Lie algebra over an algebraically closed field of characteristic 0. *O. H. Kegel*